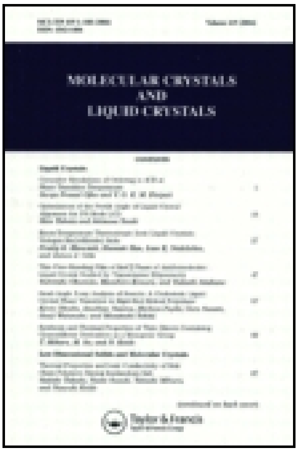

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Excitation of Rayleigh-Benard Convection in Homeotropic Nematic Liquid Crystal by a Gaussian Laser Beam from Above

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We theoretically studied the Rayleigh-Benard convection in homeotropic nematic liquid crystal cells due to the absorption of a Gaussian light beam, which penetrates the cell normally from above. It was shown that a convection of hydrodynamic motion in nematic liquid crystals can be induced as a result of absorption Gaussian beam and consequently, can lead to hydrodynamic motion and significant director reorientation in liquid crystal cells. The results can be easily measured experimentally.

Keywords Gaussian light beam; nematic liquid crystal; rayleigh-benard convection; thermal anisotropy

Introduction

Convection in a thin horizontal layer of an isotropic fluid heated from below is well known as Rayleigh - Benard convection (RBC) [1]. When the fluid is a nematic liquid crystal (NLC), this phenomenon is altered in interesting ways. In the case of thermal convection, instability mechanism for horizontal nematic samples is different from those obtained in homogeneous isotropic fluid layers [2]. The anisotropic properties and its sensibility to external forces (as magnetic or electric field) have allowed the observation of many interesting dynamical phenomena in NLC [3]. Because of the anisotropy of the liquid crystal, orientation deformation reflects back not only on the flow (viscosity anisotropy) but also on the heat transfer [4].

The axis parallel to the average orientation is called the director $\hat{n}$ in NLC. When the director is perpendicular to the horizontal boundaries it is called homeotropic. In the case of a horizontal homeotropic cell, heated from above a linear instability exists due to coupling between the flow induced by the buoyancy forces and the distortion of the

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initial director orientation. This coupling, added to the anisotropic thermal conductivity of materials, yields a heat focusing effect. Consequently, there is a dramatic reduction of the onset of convection below that of an isotropic fluid [5,6]. When the homeotropic layer is heated from below, this coupling gives rise to an oscillatory instability and convection should start roughly at the same Rayleigh number as in the isotropic case. In this case, the usual Rayleigh - Benard destabilization due to a thermally induced density gradient is opposed by the stiffness of the director field, which is coupled to and distorted by any flow. Relaxation times of the director field are much longer than thermal relaxation times, which as a consequent director fluctuations and temperature/velocity fluctuations will be out of phase as they grow in amplitude [7,8]. Experimental and theoretical investigations show that regular convective motions might be exited in a NLC due to the absorption of laser radiation with a spatially periodic intensity distribution. It was also shown that hydrodynamic motions due to the absorption of light with periodic laser intensity lead to a reorientation of the director in NLC [9–12].

This work is devoted to the theoretical study of the excitation of hydrodynamic convection in a horizontal homeotropic sample with hard anchoring condition at the cell walls that is induced by the absorption of laser radiation with a Gaussian intensity distribution from above. Here, we will study velocity and director reorientation variations due to hydrodynamic motion.

Equations of Nematic Hydrodynamic

In contrast to isotropic fluids, which the Navier-Stokes equation describes the RBC [13,14], in NLC molecular director reorientation is coupled to velocity, so the coupled equations of Navier–Stokes, molecular field and temperature diffusivity should be solved, simultaneously [4]. Here, we will use the Boussinesq approximation, which ignores the thermal variations of the physical parameters else than the thermal dependence of the density in buoyancy force. The incompressibility condition will be used in calculations which implies $\nabla, v = 0$ where v is the flow velocity.

Navier-Stokes Equation

We consider a horizontal layer $0 \geq Z \geq L$ of homeotropically oriented NLC (unperturbed director $n_0 = e_z$) in cylindrical coordinate system. The layer is in the gravitational field with $g = -ge_z$ and absorbs Gaussian incident light from above. Since the symmetry of the heat diffusivity and director orientation around the z-axis $\frac{\partial}{\partial \varphi}$ and v_φ are zero (v is the velocity of hydrodynamic motion). The excited hydrodynamic motions are described by Navier-Stokes equation in the form of

$$\rho\left(\frac{\partial v}{\partial t} + (\nabla, v)v\right) = -\nabla p + f_{\text{vis}} + f_{\text{ext}} \tag{1}$$

The i component of f_{vis} is $f_i = (\frac{\partial \sigma_{ij}}{\partial x_j})$ where σ_{ij}, the viscose stress tensor in NLC, is:

$$\sigma_{ij} = \alpha_1 n_i n_j n_k n_m d_{km} + \alpha_2 n_j N_i + \alpha_3 n_i N_j + \alpha_4 d_{ij} + \alpha_5 n_j n_k d_{ki} + \alpha_6 n_i n_k d_{kj} \tag{2}$$

n is the director unit vector and $\alpha_1 \ldots \alpha_6$ are the Leslie coefficients.

$$N_i = \frac{dn_i}{dt} + \frac{1}{2}(n \times w)_i \tag{3}$$

$$d_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{4}$$

d_{ij} is the velocity–gradient tensor, N is the rate of change of the director with respect to the immobile background fluid and w is angular velocity. We write these variables in the form, $P = P_o + P'$, $\rho = \rho_0(1 - \beta T)$, where, P_0, and ρ_0 are the unperturbed pressure, ($P_0 = -\rho_0 g z$) and density; P', v', are perturbations, and β is the volume expansion coefficient of the liquid crystal. The r- and z-components of Navier-Stokes equation after linearization will be, respectively:

$$\rho_0\left(\frac{\partial v_r(r, z, t}{\partial t}\right) = \alpha_4 \frac{\partial^2 v_r(r, z, t)}{\partial r^2} + \alpha_2 \frac{\partial^2 n_r(r, z, t)}{\partial z \partial t} + \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6)\frac{\partial^2 v_r(r, z, t)}{\partial z^2} \tag{5}$$

$$\rho_0\left(\frac{\partial v_z(r, t}{\partial t}\right) = \rho_0 T(r, z, t)\beta g + \left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\left(0.5(\alpha_2 + \alpha_4 + \alpha_6)\frac{\partial v_z(r, t)}{\partial r}\right.$$
$$\left. + 0.5(-\alpha_3 + \alpha_4 + \alpha_6)\frac{\partial v_r(r, z, t)}{\partial z}\right) + a_3\left(\frac{\partial^2 n_r(r, z, t)}{\partial r \partial t} + \frac{\partial n_r(r, z, t)}{\partial t}\frac{1}{r}\right) \tag{6}$$

Thermal Conductivity Equation

Because of the thermal anisotropy of the NLC, the heat flux q_i is described by a second - order symmetric tensor relating to the temperature gradients $q_i = -k_{ij}\frac{\partial T_i}{\partial x_j}$ [4]. Thermal conductivity has the form:

$$\rho_o c_p\left(\frac{\partial T}{\partial t} + v, vT\right) = -\nabla_q + \sigma_{ij}(\nabla v) + Q \tag{7}$$

With the used variables density ρ_0, pressure p, velocity v, temperature T, c_p specific heat, Q heat source with form $\frac{p}{2\pi\alpha^2}\exp[\frac{-r^2}{2\alpha^2}]\exp[-\alpha_\perp z]$, p light power, a spot size, $\alpha_\perp$ perpendicular cell absorption coefficient, and q heat flux vector.

$$q_i = -k_{ij}\frac{\partial T_j}{\partial_{xj}} \tag{8}$$

$$k_{ij} = k_{iso}\delta_{ij} + k_a\left(n_i n_j - \frac{\delta_{ij}}{2}\right) \tag{9}$$

$$k_{iso} = \frac{k_{||} + k_\perp}{3} \tag{10}$$

$$k_a = k_{||} + k_\perp \tag{11}$$

k_{ij} is the thermal conductive tensor, k_{iso} and k_a are isotopic and anisotopic thermal conductivities and $k_{||}$, $k_\perp$ are parallel and perpendicular thermal conductivities, respectively. The temperature equation after linearization will be in the form of:

$$\sigma_o c_p \left(\frac{\partial T(r,z,t)}{\partial}\right) + v_r(r,z,t)\frac{\partial T(r,z,t)}{\partial r} + v_z \frac{\partial T(r,t)}{\partial z} = \frac{1}{r}k_\perp \left(\frac{\partial T(r,z,t)}{\partial r}\right)$$
$$+ k_\perp \left(\frac{\partial^2 T(r,z,t)}{\partial r^2}\right) + k_{||}\frac{\partial^2 T(r,z,t)}{\partial z^2} + \alpha_\perp \frac{p}{2\pi a^2}\mathrm{Exp}\left[\frac{-r^2}{2a^2}\right][-a_j z] \qquad (12)$$

Torque Balance Equation

The balance of the torques exerted on the molecule indicates the director reorientation. These forces are the viscose hydrodynamical force and the Frank elasticity force. The torque balancing equation is:

$$(n \times f)_i + e_{ijm}n_m \left[\frac{\partial F}{\partial_{nj}} - \frac{\partial}{\partial x_k}\frac{\partial F}{\left(\frac{\partial n_j}{\partial x_k}\right)}\right] = 0 \qquad (13)$$

$$f_\downarrow i = (\gamma_\downarrow 1 N_\downarrow i + \gamma_\downarrow 2 d_\downarrow ij) \qquad (14)$$

$$N_i = \frac{dn_i}{dt} + \frac{1}{2}(n \times w)_i \qquad (15)$$

$$\gamma_1 = (-\alpha_2 + \alpha_2) \qquad (16)$$

$$\gamma_2 = (\alpha_2 + \alpha_2) \qquad (17)$$

$$F = k_1(\nabla, n)^2 + k_2(n, \nabla \times n)^2 + k_3(n \times \nabla \times n)^2 \qquad (18)$$

where f is the hydrodynamical force, F is the Frank free energy and $k_1 \ldots k_2$ are the Frank elastic constants. The torque balance equation after linearization is as follows:

$$\gamma_1 \dot{n}_r(r,z,t) - \frac{1}{2}\gamma_1\left(\frac{\partial v_r(r,z,t)}{\partial_z} - \frac{\partial v_z(r,t)}{\partial_r}\right) + \frac{1}{2}\gamma_2\left(\frac{\partial v_r(r,z,t)}{\partial_z} + \frac{\partial v_z(r,t)}{\partial_r}\right)$$
$$+ k_1\left(-\frac{2n_r(r,z,t)}{r^2} + \frac{\partial^2 n_r(r,z,t)}{\partial r^2}\right) + k_3\frac{\partial^2 n_r(r,z,t)}{\partial z^2} = 0. \qquad (19)$$

Numerical Calculations

We will solve coupled equations 5, 6, 12, and 19 with the following boundary conditions: consider the NLC at the rest initially, symmetric around the z-axis with hard anchoring conditions at the both NLC cell walls and with no effect at the infinite;

$$T(r,z,t) = v_r(r,z,t) = v_z(r,z,t) = \delta n_r(r,z,t) = 0 \qquad r = \infty \qquad (20)$$

$$v_r(r,z,t) = \delta n_r(r,z,t) = v_z(r,z,t) = T(r,z,t) = 0 \qquad z = 0, L \qquad (21)$$

$$\frac{\partial}{\partial_r}v_r(r,z,t) = \frac{\partial}{\partial_r}v_z(r,z,t) = \frac{\partial}{\partial_r}\delta n_r(r,z,t) = \frac{\partial T(r,z,t)}{\partial_r} = 0 \qquad r = 0. \qquad (22)$$

For the numerical estimates, the following parameters were used for the NLC 5CB:

$$\rho_o = 1.0215\frac{g}{\mathrm{cm}^3},\ \alpha_2,\ \alpha_2,\ \alpha_4,\ \alpha_6 \text{ are respective}$$
$$-0.77,\ -0.042,\ 0.634,\ -0.1840\frac{dynes}{\mathrm{cm}^2}$$

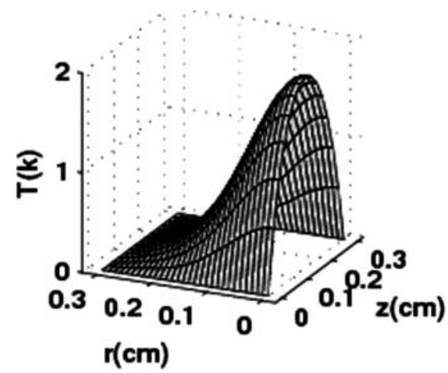

Figure 1. Temperature distribution versus r, z with $p = 7\,\text{mW}$, $\alpha_\perp = 3\,\text{cm}^{-1}$, $a = 1\,\text{mm}$.

$$\rho_o c_p \approx 1,\ g = 980.668\frac{\text{cm}}{s^2},\quad k_\perp = 6.19 * 10^{-4}\frac{\text{cm}^2}{s},$$

$$k_{||} = 1.14 * 10^{-3}\frac{\text{cm}^2}{s}, \alpha_\perp = 3\,\text{cm}^{-1},$$

$$p = 7\,\text{mW},\ a = 1\,\text{mm},\quad k_1 = 5.95\times 10^{-7}\,dynes,\quad k_3 = 7.86\times 10^{-7}\,dynes,$$

$$L = 3\,\text{mm}, t = 60s$$

Result and Discussion

It was found that the temperature follows the Gaussian light beam profile in the r- direction but with larger spot size in accordance with other reported experiments (Fig. 1), its maximum is nearly at the middle of the cell due to the considered boundary condition.

Figure 2 indicates radial hydrodynamic flow velocity in r- and z-directions using the mentioned NLC parameters. The flow follows the temperature gradient in r direction and changes its direction at the middle of the cell in z direction that illustrates the existence of

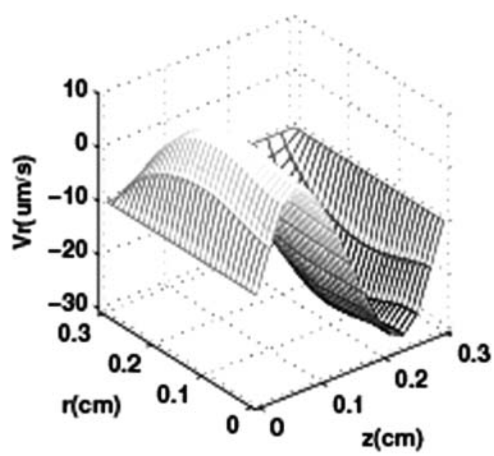

Figure 2. Flow velocity in r direction versus r, z with $p = 7\,\text{mW}$, $\alpha_\perp = 3\,\text{cm}^{-1}$, $a = 1\,\text{mm}$.

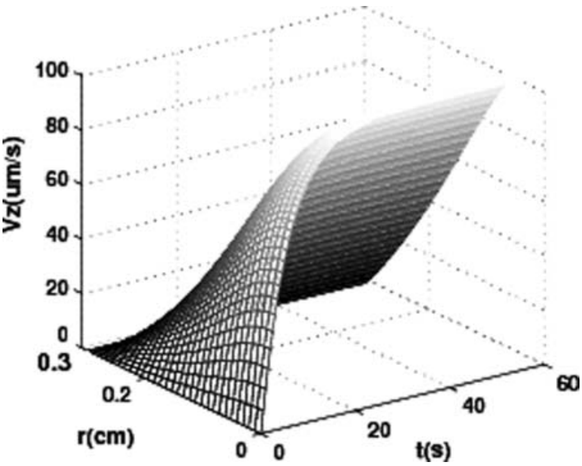

Figure 3. Flow velocity in z direction versus *r, t* with $p = 7\,\text{mW}$, $\alpha_{\perp} = 3\,\text{cm}^{-1}$, $\alpha = 1\,\text{mm}$.

circular rolls in NLC due to RBC. Coupling between director and velocity can reorient the molecular director in NLC (Fig. 2). Figure 3 shows v_z variations. It is seen that the velocity increases with time at first but after some heating, reaches a saturated magnitude.

We can see the director reorientation of the molecules follows the temperature gradient in the r-direction. In the z- direction we have a smooth curve with a maximum at the middle of the cell thickness except near the boundaries, which can be a result of the anchoring force (Figs. 4, and 5). It is shown that the director has the temperature gradient profile shape

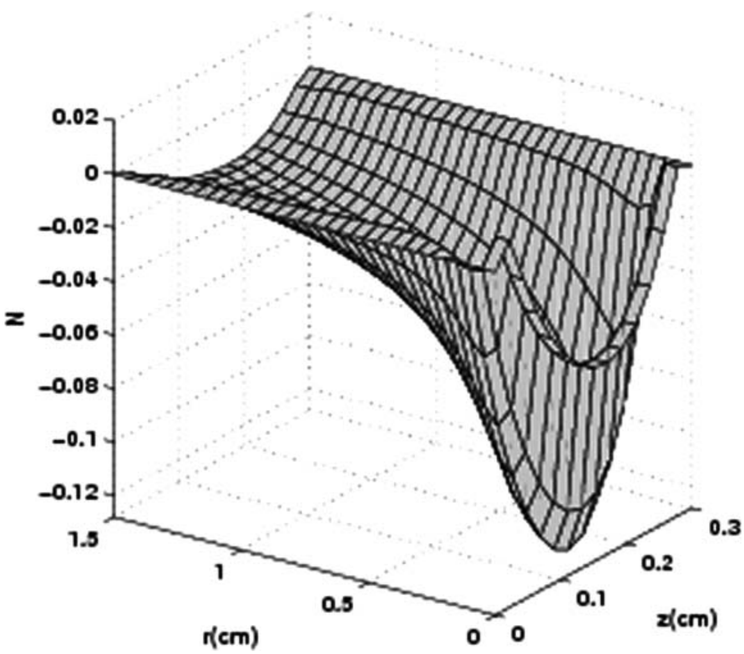

Figure 4. Director reorientation versus *r, z* with $p = 7\,\text{mW}$, $\alpha_{\perp} = 3\,\text{cm}^{-1}$, $\alpha = 1\,\text{mm}$.

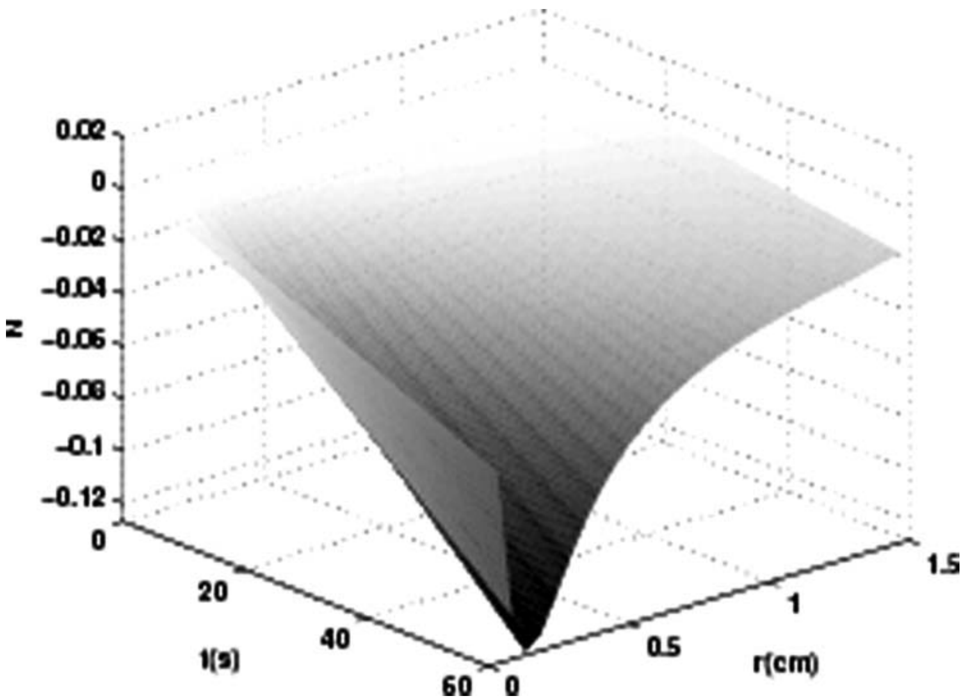

Figure 5. Director Reorientation versus *r*, *t* with $p = 7\,\mathrm{mW}$, $\alpha_{\perp} = 3\,\mathrm{cm}^{-1}$, $\alpha = 1\,\mathrm{mm}$.

with a remarkable reorientation. This reorientation can be considered as one of the largest director reorientations in NLC and is comparable with the director reorientation due to the giant optical nonlinearity [15].

Conclusion

We studied the Rayleigh - Benard Convection due to the absorption of a Gaussian laser profile. It was shown that the instability can happened in a NLC, which is heated from above which has no similar isotropic fluids. This effect is a result of the coupled anisotropy in the medium. The calculation shows that an approximately circular roll can exist in this case, but decaying occurs with distance from the light intensity center. Coupling between the molecular director, velocity and temperature leads to significant director reorientation. This reorientation can be one of the largest director reorientations seen in NLC.

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